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Wave Propagation in an Ion Beam Plasma System

by

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Abstract

The spatial evolution of a velocity - or density - modulated ion beam is calculated for stable and unstable ion beam plasma systems, using the linearized Vlasov - Poisson equations. The propagation properties are found to be strongly dependent on the form of modulation. In the case of velocity modulation, the perturbation grows initially and then shows a periodic change of amplitude along the beam, while in the case of a density modulation only an instability causes growth. The findings are in agreement with experimental results obtained by Sato et al. (1977).

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CONTENTS

1. INTRODUCTION

The propagation of waves in a plasma traversed by an ion beam has recently received much attention because of its importance in connection with plasma instabilities and heating. Regions of instability for the ion-beam instability have been calculated by several authors, e.g., Harrison (1962), Stringer (1964), Fried and Wong (1966), and Michelsen and Prahm (1971). Several investigators have reported experiments on this instability in Q-machines, Baker (1972 and 1973), and Christoffersen and Prahm (1973), and in double-plasma devices, Grésillon and Doveil (1975), Kiwamoto (1974), Taylor and Coroniti (1972), and Fujita et al. (1975). The propagation of both short pulses and continuous waves has been studied in such systems, and the system has been classified as unstable in cases where the pulse or the wave amplitude increased away from the exciter. In an experiment performed in an ion-beam plasma system generated in a double-plasma-operated Q-machine, Sato et al. (1975 and 1977) found spatial growth although the plasma was predicted to be stable according to theory. This growth was explained by linear theory for beam bunching, which occurs in the case of velocity modulation, both for the ballistic contribution as well as for the collective modes. Ion waves in double-plasma devices are normally excited by velocity modulation of an ion beam, but also grid excitation of waves in other machines often gives rise to perturbations in the ion velocity distribution, which may be characterized as velocity modulation, or as a combination of velocity and density modulation, Christoffersen (1971) and Grésillon (1971).

Recently, we performed calculations on pulse propagation in an ion beam plasma system, Michelsen et al. (1976). In the present paper we report on analytical and numerical calculations of wave propagation in stable and unstable ion beam plasma systems. The calculations, based on the Vlasov-Poisson equation, were motivated by the interesting measurements of Sato et al. (1977). Therefore, to obtain the best agreement with the experiment, the equations were solved as a boundary value problem. The theory is similar to that applied by Christoffersen et al. (1974), but the analysis is performed in a simpler way and also extended to include unstable systems. For the reasons mentioned above, we especially concentrated the calculations on pure velocity modulation and for comparison also included pure density modulation.

The theory is summarized in Sec. 2, the numerical calculations and results are given in Sec. 3, while Sec. 4 contains a discussion and conclusion.

2. THEORY

This section summarizes the theory used for the calculations. We describe the motion of the ions by their linearized Vlasov equation, and that of the electrons by a massless isothermal fluid. Ions and electrons are coupled through the assumption of quasi-neutrality, i.e., we restrict our considerations to long waves $(k\lambda_D)^2 \ll 1$ (λ_D is the Debye length). We consider a one-dimensional situation. Thus, our basic equations are

$$\frac{\partial f(x,v,t)}{\partial t} + v\,\frac{\partial f(x,v,t)}{\partial x} = \frac{e}{M}\,\frac{\partial \phi(x,t)}{\partial x}\,\frac{\partial f_o(v)}{\partial v} \qquad (1)$$

$$T_e \frac{\partial n(x,t)}{\partial x} = n_o e \frac{\partial \phi(x,t)}{\partial x} \tag{2}$$

and

$$n(x,t) = \int_{-\infty}^{\infty} f(x,v,t)\, dv, \tag{3}$$

where $f_o(v)$ is the zero-order distribution function, $f(x,v,t)$ is the perturbed distribution function, $\phi(x,t)$ is the perturbed electrical potential, n_o is the zero-order density given by $n_o = \int_{-\infty}^{\infty} f_o(v)dv$, $n(x,t)$ is the perturbed density, e is the charge, M is the mass of the ions and T_e is the electron temperature in energy units. Equations (1)-(3) describe, for instance, wave propagation along the magnetic lines in a Q-machine plasma (Jensen, 1976).

As plasma-wave experiments are usually boundary-value problems, we studied the propagation of ion-acoustic waves excited at the boundary of a semi-infinite plasma. Thus equations (1)-(3) are solved as a boundary-value problem with the following boundary values:

$$f(x=o,v,t) = g(v)\exp(-i\omega_o t),$$

$$n(x=o,t) = \int_{-\infty}^{\infty} g(v)\exp(-i\omega_o t)dv = \eta\exp(-i\omega_o t).$$

Additionally, we assume that $f_o(v)$ and $g(v)$ is zero for $v \leq o$, i.e., we need not specify the boundary values at $x \to \infty$. To solve equations (1)-(3) we can then use a Laplace transform in space, (Nielsen, 1969, Pécseli, 1974, and Christoffersen et al., 1974).

By applying the Laplace transform in space and the

Fourier transform in time, i.e.,

$$f(k,\omega) = \int_{-\infty}^{\infty}\int_{0}^{\infty} e^{i(\omega t-kx)} f(x,t)dx\ dt,$$

we obtain

$$n(\omega,k) = \frac{1}{ik}\ 2\pi\ \delta(\omega_o-\omega)\ M(\frac{\omega}{k})\ ,$$

where

$$M(\frac{\omega}{k}) = \frac{\frac{\omega}{k}\int_{-\infty}^{\infty}\frac{g(v)}{v-\omega/k}dv}{1-\frac{c_e^2}{n_o}\int_{-\infty}^{\infty}\frac{f'_o(v)}{v-\omega/k}\,dv} + \eta \tag{4}$$

and

$$c_e = (T_e/M)^{\frac{1}{2}}.$$

The inverse transform is

$$n(x,t) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\exp(-i\omega t)\int_{-\infty-i\alpha}^{\infty-i\alpha}\exp(ikx)n(k,\omega)\ dk\ d\omega,$$

where α is a positive quantity assuring that the integration path in the k plane runs below all singularities in $n(k,\omega)$.

The ω-integration gives immediately

$$n(x,t) = \frac{1}{2\pi}\exp(-i\omega_o t)\int_{-\infty-i\alpha}^{\infty-i\alpha}\frac{1}{ik}\ M(\frac{\omega_o}{k})\exp(ikx)\ dk,$$

The integrands in the v-integrations in $M(\frac{\omega_o}{k})$ have poles for $v = \frac{\omega_o}{k}$. We obtain an analytic continuation of $M(\frac{\omega_o}{k})$, termed $M_b(\frac{\omega_o}{k})$, by prescribing that the v-integration path shall run below the pole. For a stable situation, the function $M_b(\frac{\omega_o}{k})$ has no poles in the complex k plane with Imk < o. For an unstable situation, $M_b(\frac{\omega_o}{k})$ has one pole $k=k_p$ for $Imk_p < 0$. We note that there are no singularities in the origin because we find from Eq. (4) that $\lim_{k \to 0} (\frac{1}{k} M(\frac{\omega_o}{k})) = 0$. We can change the k-integration-contour to the real axis and split up the integration into two contributions.

$$n_s(x,t) = \frac{1}{2\pi} \exp(-\omega_o t) \left[\int_{-\infty}^{o} \frac{1}{ik} M_b(\frac{\omega_o}{k}) \exp(ikx)\, dk \right.$$

$$\left. + \int_{o}^{\infty} \frac{1}{ik} M_b(\frac{\omega_o}{k}) \exp(ikx)\, dk \right]. \qquad (5)$$

If the plasma is unstable, we must add the residue contribution from the unstable pole. To carry out the integration in (5) we change the contour of the k-integration into the complex k-plane (Gould, 1964). That is we define a new function $M_a(\frac{\omega_o}{k})$ as the analytical continuation of $M(\frac{\omega_o}{k})$, where, in contrast to $M_b(\frac{\omega_o}{k})$, it is specified that the integration path in the v-integrations shall run <u>a</u>bove the poles. We thus have the following relation between the two functions

$$M_a^*(\zeta^*) = M_b(\zeta),$$

where the asterisk denotes the complex conjugate.

In consequence of the assumption that there are no ions with negative velocity, i.e. $f_o(v) \equiv g(v) \equiv 0$ for $v < 0$, we can replace M_b by M_a in the first integral in equation (5), because M_b and M_a are equal on the real negative k-axis. As M_a is analytic for a stable case in the upper imaginary k-half-plane, we can change the integration contour to run just above the positive real k-axis as shown in Fig. 1. Because the integration along the half-circle Γ is zero, we can reduce equation (5) to one integral running from 0 to ∞. If the plasma is unstable, we have to add the residue of the unstable pole in k_p, and the residue of the function $M_a(\zeta)$ in k_p^* (Fig. 1). We then find that the total expression for the perturbed density, after having omitted the time dependence, can be written:

$$n(x) = \frac{1}{\pi}\int_0^\infty \frac{1}{k}\mathrm{Im}\{M_b(\frac{\omega_o}{k})\}\exp(ikx)\,dk$$
$$+ \mathrm{Res}\{\frac{1}{k}\,M_a(\frac{\omega_o}{k})\exp(ikx)\}\Big|_{k=k_p^*} \qquad (6)$$
$$+ \mathrm{Res}\{\frac{1}{k}\,M_b(\frac{\omega_o}{k})\exp(ikx)\}\Big|_{k=k_p},$$

where Im{} and Res{} stand for the imaginary part and the residue of the quantity in the braces. For a stable case, only the first term makes a contribution. n(x) in Eq. 6 is a complex quantity, where the modulus and the phase respectively give the amplitude and the phase of the perturbed density.

In the calculation of $n(x)$ use is made of a zero-order distribution function, which consists of a sum of two drifting Maxwellians:

$$f_o = f_i + f_b$$
$$= n_o \frac{1}{\sqrt{\pi}}(n_i \frac{1}{c_i} \exp[-(\frac{v-v_i}{c_i})^2] + n_b \frac{1}{c_b}\exp[-(\frac{v-v_b}{c_b})^2]), \quad (7)$$

where the indices i and b refer to plasma ions and beam ions, respectively, $n_{i,b}$ is the relative density $(n_i+n_b=1)$, $c_{i,b} = (\frac{2T_{i,b}}{M})^{1/2}$ is the thermal speed (beam and plasma ions are of the same kind), and $v_{i,b}$ is the drift velocity. Using an $f_o(v)$ just described and a $g(v)$, which also consists of Maxwellians, the plasma dispersion function can be used in our calculations. Only an approximate solution can be obtained with these distributions, since $f_o(v) \neq o$ and $g(v) \neq o$ for $v \lesssim o$. However, the approximation is good for Maxwellians with drift velocities greater than 2 c_i. The ion distribution in a single-ended Q-machine is approximately described by a drifting Maxwellian, Andersen et al. (1971).

3. NUMERICAL RESULTS

In this section the propagation properties of ion-acoustic waves in the beam plasma system are studied by numerical calculation of the density perturbation given in Eq. (6) using the distribution function in (7), (Fig. 2a). Special attention is paid to the difference between the propagation of waves generated by a velocity modulation and a density modulation of the beam. Velocity modulation of the beam is obtained by using a velocity distribution $g(v) = f_b(v+\tilde{v})-f_b(v)$, i.e., $g(v)$ is the difference between two Maxwellians separated by $\tilde{v}$, Michelsen et al. (1976). In all the calculations $\tilde{v} = 0.05\ c_b$; $g(v)$ for this case is shown in Fig. 2(b). Density modulation of the beam is realized by using a $g(v)$ that is proportional to $f_b(v)$. In all our calculations the drift velocity of the background plasma v_i is 2 c_i to satisfy the assumption used in Sec. II. The result of the calculations of the wave propagation is shown in Figs. 3-6, where the perturbed density $n(x)$ (solid line) is plotted versus the normalized distance $x\omega/c_i$. The dashed line in the figures indicates the wave amplitude. The ordinate is in arbitrary units.

Figure 3 shows the propagation properties of waves in the case of density modulation for decreasing values of v_d ($\equiv v_b-v_i$). The parameters of the zero-order distribution function are $T_b = T_i$, $n_b = n_i$ and $v_d = 7c_i$, $4.1c_i$, $4c_i$ and $3c_i$, respectively.

Further, $T_e = 5\ T_i$, which means that the collective beam modes will dominate the free-streaming contribution after a few wavelengths. In Figs. 3a and b the system is stable and the wave patterns show a periodical oscillation in the amplitude superimposed by a monotonic damping. The amplitude oscillation is interpreted as caused by the beating between the fast and the slow ion-beam mode (Sato et al. 1977). The phase velocities of the fast and slow beam mode, v_f and v_s respectively, are approximately given by

$$v_{f,s} = v_b \pm n_b^{\frac{1}{2}}\ c_e\ (1 + 3\ T_b/T_e)^{\frac{1}{2}}\ . \qquad (8)$$

In accordance with this expression, the pitch length in the interference pattern decreases with decreasing v_b. Since the slow mode is more strongly damped than the fast mode, the interference becomes weaker for increasing distance. This effect becomes more pronounced for decreasing v_b, because the difference in the damping of the fast and slow mode becomes larger. Close to the exciter, the waves have an "average" phase velocity close to v_b, but far away the phase velocity approaches v_f. Note, however, that the phase of the wave shows abrupt changes where the amplitude has minima. Figures 3c and d show unstable situations, Michelsen and Prahm (1971). Again we have the amplitude oscillations but after a few wavelengths the amplitude grows exponentially and here the phase velocity approaches the velocity of the unstable mode, i.e.: the velocity of the minimum in $f_o(v)$ (Jensen et al. 1974).

Figure 4 shows the corresponding curves (same parameters as in Fig. 3) obtained with a velocity modulation of the beam. We notice that the main difference between the curves in Fig. 3

and those in Fig. 4 is that for a velocity modulation the waves grow from zero at the exciting point; but after a few wavelengths no significant differences are seen between corresponding curves in Figs. 3 and 4.

Figures 5 and 6 show propagation properties of waves excited by velocity modulation. The parameters of the distribution functions are chosen similar to those in the experiments of Sato et al., 1977. In Fig. 5 we show the wave pattern for varying beam velocities, v_b. When an ion beam is accelerated to the velocity v_b it is adiabatically cooled, and its temperature T_b is determined by (Sato et al., 1977),

$$\frac{T_b}{T_i} = \frac{1 - \frac{3}{2}\left(\frac{v_d}{c_i}\right)^{\frac{1}{2}}}{2\left(\frac{v_d}{c_i}\right)^2} \quad . \tag{9}$$

These estimated temperatures are not realistic, and Sato et al. (1977) found in experiments that the actual temperature is approximately 1.5 - 2.5 times larger. We chose an even larger factor ($\sim$5), because, with the very low beam temperature estimated by (9), the M-function (4) becomes very narrow, and it is difficult to compute the integral in (6) sufficiently accurately. We note (Fig. 5) that the tendency for the perturbed density to split up into a perfect interference pattern is weakened when v_b decreases. For $v_b = 3.5\ c_i$ the amplitude damps away monotonically without any oscillations. The wavelength λ and the pitch length l of the periodic amplitude variation decrease as v_b decreases. In Fig. 6 the wave pattern is shown with n_b as a parameter. As n_b decreases, l increases (e.g. eq. 8) and the periodic change of the amplitude gradually disappears. For small values of n_b, the perturbations grow initially and then they damp away monotonically after reaching maximum. The phase velocity is close to v_f when the wave is strongly damped. A comparison between the measurements by Sato et al. (1977) and our corresponding calculations shows an excellent agreement.

4. DISCUSSION AND CONCLUSION

We have presented analytical and numerical calculations of the spatial evolution of density waves in an ion beam plasma system. The calculations are based on the linearized Vlasov equation, which is solved as a boundary value problem in semi-infinite plasma. Motivated by the experiments of Sato et al. (1975 and 1977), we investigated the cases where the wave is excited both by a pure velocity and by a pure density modulation of the beam, in stable as well as in unstable systems. In the case of the velocity modulation, initial growth and subsequent amplitude oscillation are found for the stable situation (Fig. 4). This behaviour was also found in the experiment of Sato et al. (1977) where the waves were excited by modulating the velocity of the ion beam in a DP-type plasma. By using the data from their experiment in our calculations, we found excellent agreement between our results (Figs. 5 and 6) and their measurements, except that in our case the damping is somewhat stronger. This is caused by the fact that we chose a higher value of the beam temperature (see Sec. 3).

The amplitude oscillation is explained by the beating between the fast and the slow ion-beam-mode, so-called beam bunching. The same mechanism causes the amplitude oscillation in the case of density modulation of the beam for the stable situation (Figs. 3a and b). However, there is an important difference:

in the case of velocity modulation of the beam the wave initially grows from zero amplitude at the exciting position, while in the case of density modulation there is only a very small initial growth (Fig. 3a). We attribute the small growth in the latter case to the influence of the background plasma, as no initial growth was found when only a beam was present.

The phase velocity of the wave is close to the beam velocity as would be expected from Eq. (8). However, since the damping of the slow mode is stronger than that of the fast mode, a tendency that becomes clearer for decreasing v_b respectively n_b, (Figs. 4, 5 and 6), only the fast mode survives some distance from the exciter. The phase velocity then approaches that of the fast mode. From wave patterns like those shown above, we see that it will be very difficult to measure the exponential (Landau) damping of the waves, while the phase velocities of the two beating modes (deduced from the wavelength and the pitchlength) are in good agreement with those calculated from the linear dispersion relation. Calculations of the propagation of density pulses in a similar system also showed agreement with the linear dispersion relation, i.e., the initial pulse splits up into three pulses propagating with the speed of the background ion mode, the slow and the fast ion beam mode, respectively (Rasmussen, 1977).

It has often been claimed (e.g. Jensen, 1976) that the propagation properties of ion waves depend strongly on the distribution function in the perturbation at the boundary (i.e., g(v)) when $T_e/T_i \lesssim 3$, and the ballistic contribution dominates the collective modes. In our case we also see that the shape of g(v) influences the wave propagation, even if $T_e/T_i = 5$, and the collective modes dominate after a few wavelengths. This result was demonstrated very clearly by investigating the propagation of density pulses (Michelsen et al., 1976). In this connection it is worth noting that a purely ballistic theory (i.e., $T_e = 0$) will also result in a wave with amplitude oscillations similar to those in Fig. 4a in the case of velocity modulation (Sato et al., 1977). We also found these oscillations by putting $T_e = 0$ in our calculations. A density modulation only gives a monotonically decaying amplitude (e.g., Jensen and Michelsen, 1972). In Figs. 4 - 6 the first pitch is a little longer than the others, i.e., the phase velocity difference between the beating waves is smaller than for the other pitches. This is caused by the contribution from the free-streaming ions in g(v), which dominates in the vicinity of the exciter. Because of the two humps of g(v) (see Fig. 2b), this contribution resembles the interference pattern from two modes with a phase velocity difference determined by the velocity difference in the two humps of g(v), and this difference is smaller than $v_f - v_s$.

When the plasma is unstable with respect to the ion-ion instability (Fig. 3c, d, 4c and d), we still find the amplitude oscillations caused by the interference between the slow and the fast beam mode close to the exciter, before the unstable mode begins to dominate. Thereafter the amplitude grows exponentially, superimposed by a decaying oscillation. This behaviour is followed by a decrease in the phase velocity. Near the exciter,where the beating between slow and fast beam mode is dominant, the phase velocity (in the maxima) is close to the beam velocity, while in the region where the growth is significant, the phase velocity approaches the velocity of the unstable mode which is <u>less</u> than the beam velocity. (In the cases treated here this phase velocity is equal to the velocity of the minimum of $f_o(v)$, i.e., $(v_b + v_i)/2$). These properties of wave propagation in an unstable ion-beam plasma system, with parameters comparable to those used here, were observed experimentally by Christoffersen and Prahm (1973). In a similar experiment by Baker (1973) no clear phase velocity change could be deduced from the wave patterns and only the unstable mode was observed. However, according to the parameters given by Baker, his system is strongly unstable. Actually, in calculations with such parameters we also find that the unstable mode dominates after a few wavelengths. From Figs. 3c, d, 4c and d the shape of g(v) is seen to be unimportant for the unstable growth, which is also clear from (6). In the case of pulse propagation, however, the shape of g(v) can affect the form of the unstable pulse (Michelsen et al. 1976). Even if g(v) has no influence on the growth of the unstable mode, one can imagine that it will have some importance for the wavepattern in the unstable situation, which in fact is seen in Figs. 3c, d and 4c, d. Further, g(v) could be chosen (or in experiments prove to be) such that the one mode is preferentially excited, e.g., the fast beam mode, and this wil dominate for several wavelengths before the unstable mode starts to grow. Conversely, the unstable mode could be excited and then only the growing mode would be seen. Actually, Pécseli (1975) has shown that, for a given development of the density perturbation, a g(v) always can be prescribed (although this is not always physically possible).

We only considered weakly unstable situations, and our results for the unstable mode are, of course, only valid until the instability reaches a nonlinear level. In experiments on the ion-ion instability (e.g. Baker, 1972, 1973; Christoffersen and Prahm, 1973; Gréllion and Doveil, 1975; Fujita et al. 1975), it has been found that the unstable wave grows initially, but is subsequently saturated due to nonlinear effects and damped; a behaviour very similar to that which can be found in the case of a velocity modulation of the beam, e.g. Figs. 6c and d. It should be emphasized that the difference can be seen from the phase velocity. In the stable case with beam-bunching, the phase velocity tends to increase and approaches the velocity of the fast beam mode, i.e. larger than the beam velocity, with increasing distance from the exciter. In the unstable case, on the other hand, the phase velocity decreases and approaches the velocity of the unstable mode, i.e. smaller than the beam velocity, with increasing distance from the exciter. However, even if this clear difference makes it relatively simple to identify an unstable mode from a stable one, if the parameters of the distribution function are known accurately enough, which is certainly not always an easy matter, the initial appearance of the beating between the stable modes as seen in Figs. 3c, d, 4c and d can make it very difficult to measure an exact growth rate of the instability.

The amplitude oscillations described above were caused by the beating between normal modes propagating in the same direction, i.e. to produce these phenomena an ion beam with a velocity somewhat greater than the speed of sound is necessary. Such a situation is found not only in beam plasma systems (DP-type plasmas), but also in single-ended Q-machines under the "electron-rich condition" where an ion beam, accelerated by the negative hot plate sheath, flows through the electrons and is absorbed by a negatively biased target. Ion acoustic waves in such a plasma are usually excited by grids, which often give rise to complicated, perturbed, distribution functions (Christoffersen, 1971). However, these may frequently be described by a combination of density and velocity modulation (Grésillon, 1971). If the ion beam temperature is much lower than the electron temperature, which it must be in the low density case since the accelerating electron sheath is rather large here, we should expect the excitation of both the fast and slow beam mode, and therefore amplitude oscillations. These were probably observed by Buzzi (1974).

Finally, we should like to point out that amplitude oscillations of plasma waves have usually been interpreted as caused by nonlinear effects. Our calculations, on the other hand, show that the same behaviour may occur as a mere superposition of linear normal modes, in agreement with experimental observations (e.g., Sato et al. 1975, 1977; Grésillon and Doveil, 1975).

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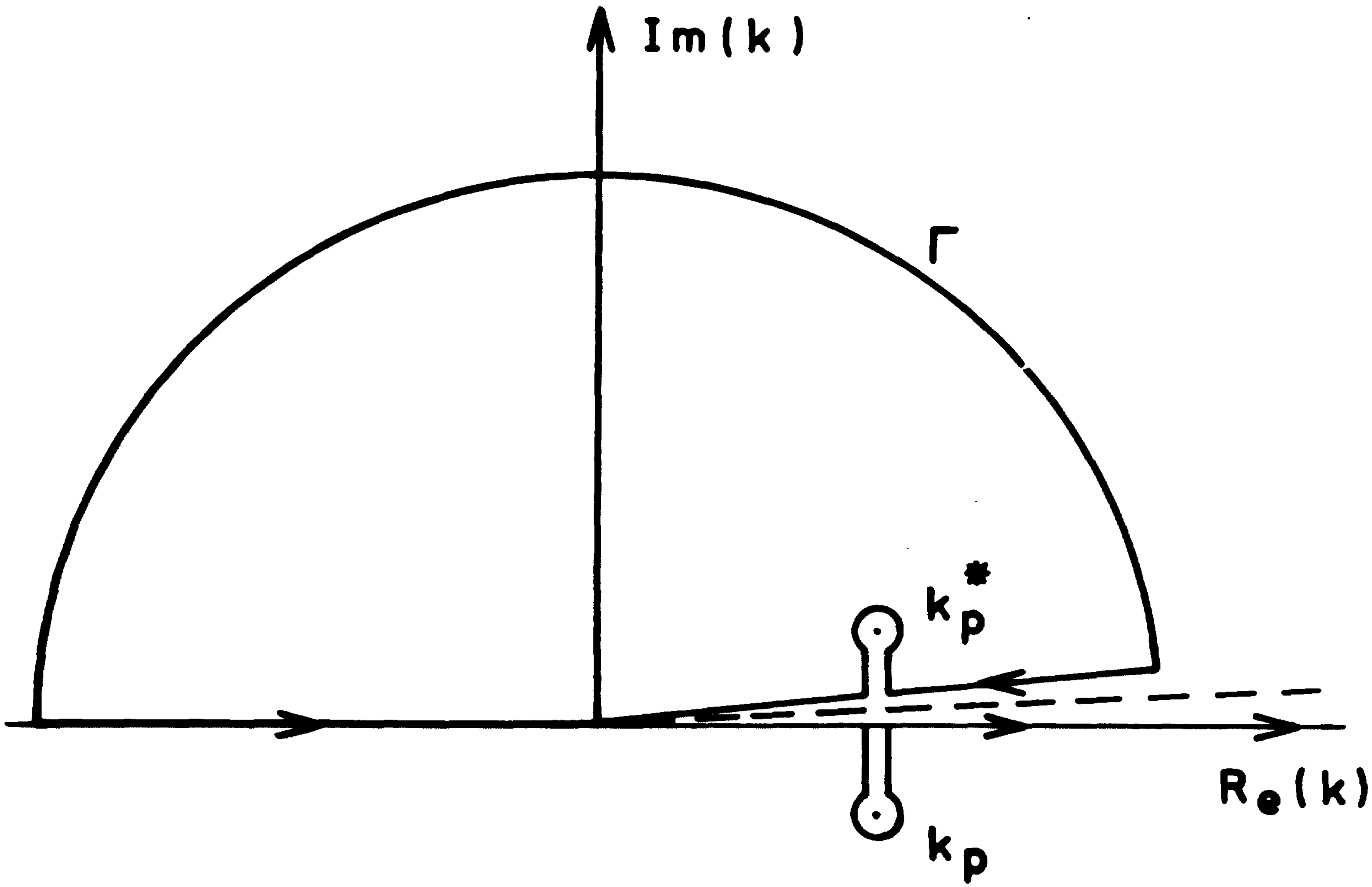

Fig. 1. Integration contour. k_p is the unstable pole, and the dashed line is a branch cut.

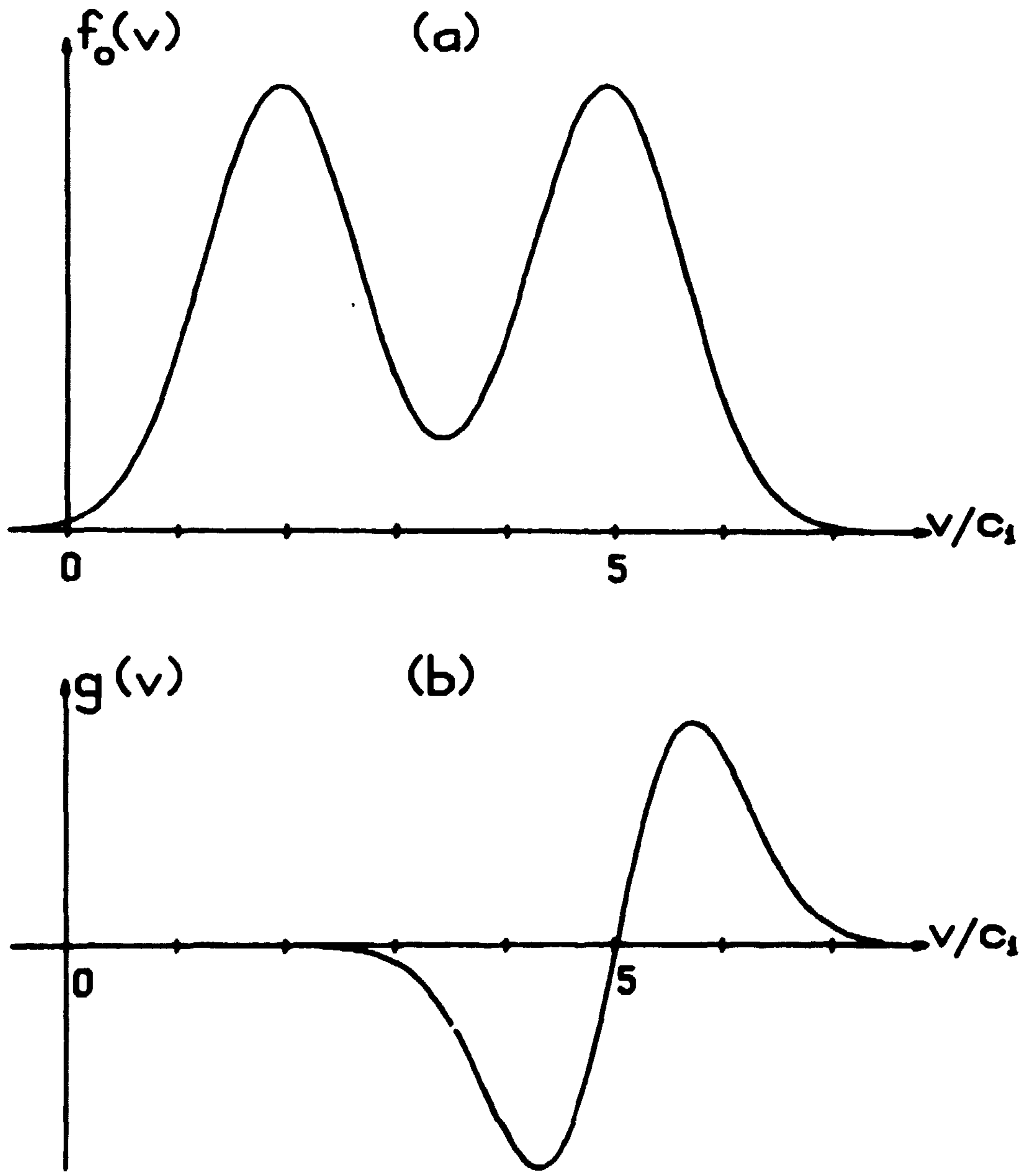

Fig. 2. (a) The zero-order ion velocity distribution function.
(b) The perturbed distribution function at the boundary in the case of velocity modulation.

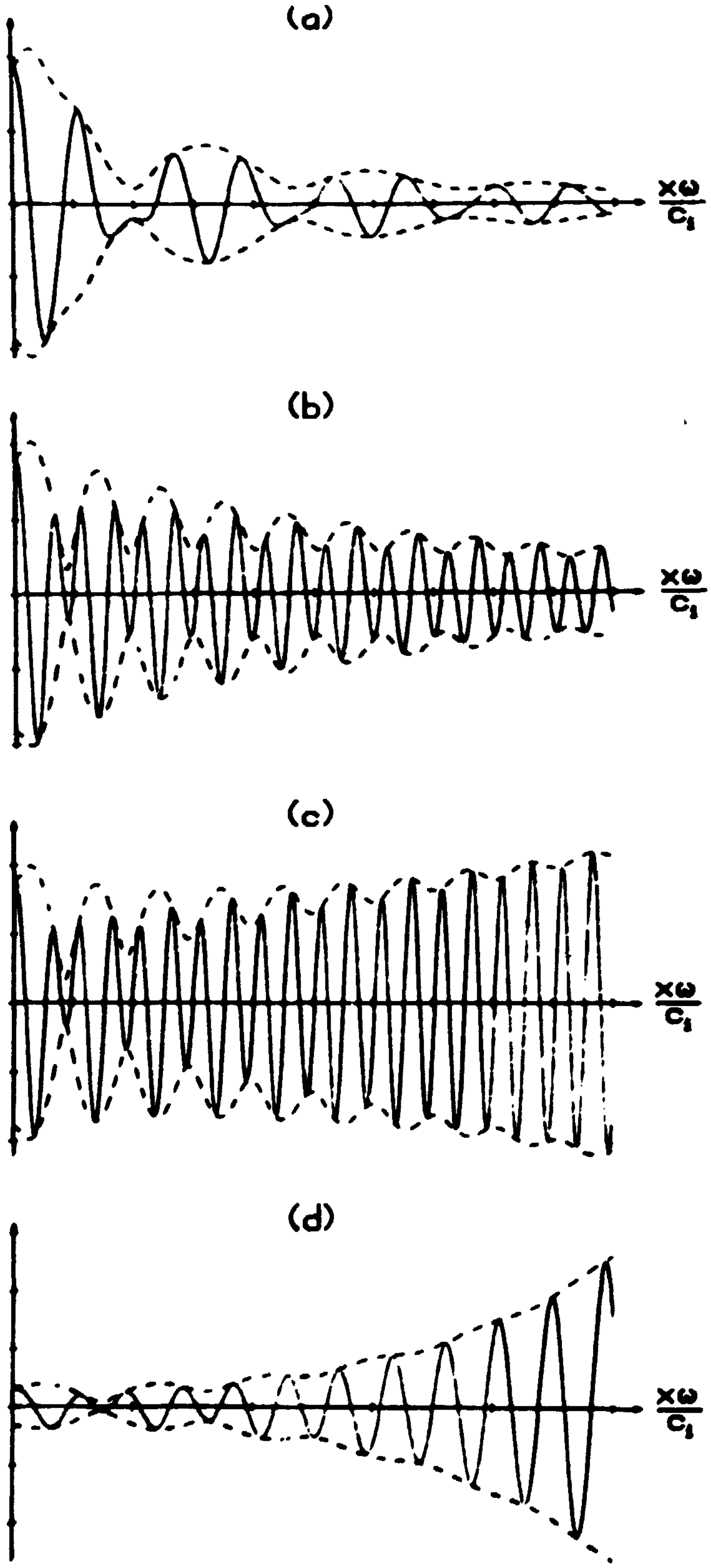

Fig. 3. Wave propagation for density modulation. $T_b = T_i$, $T_e = 5\ T_i$, $n_i = n_b$, $v_i = 2c_i$, abscissa = $50x\omega/c_i$ pr. division. Stable cases: (a) $v_b = 9c_i$ and (b) $v_b = 6.1\ c_i$. Unstable cases: (c) $v_b = 6.0\ c_i$ and (d) $v_b = 5\ c_i$.

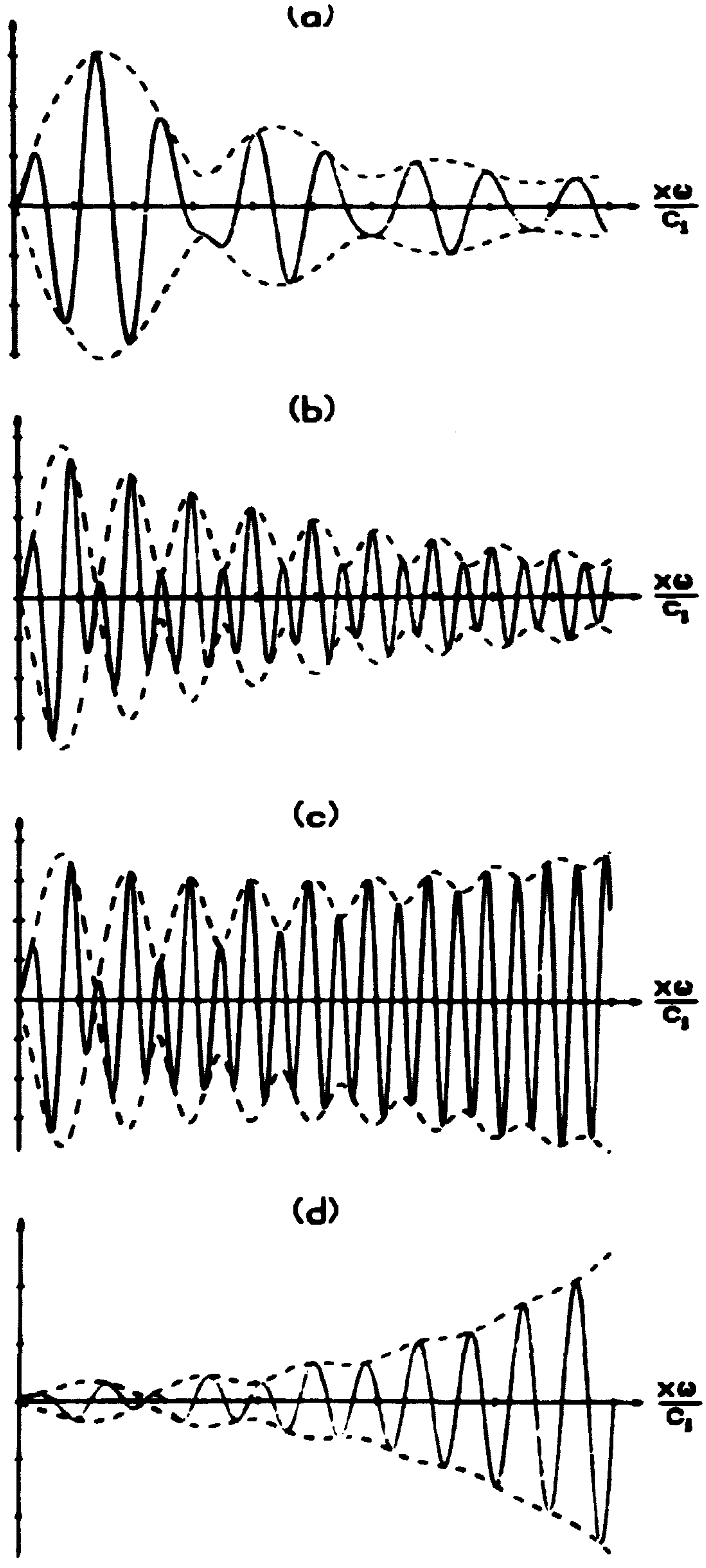

Fig. 4. Wave propagation fc velocity modulation. All parameters as in Fig. 3.

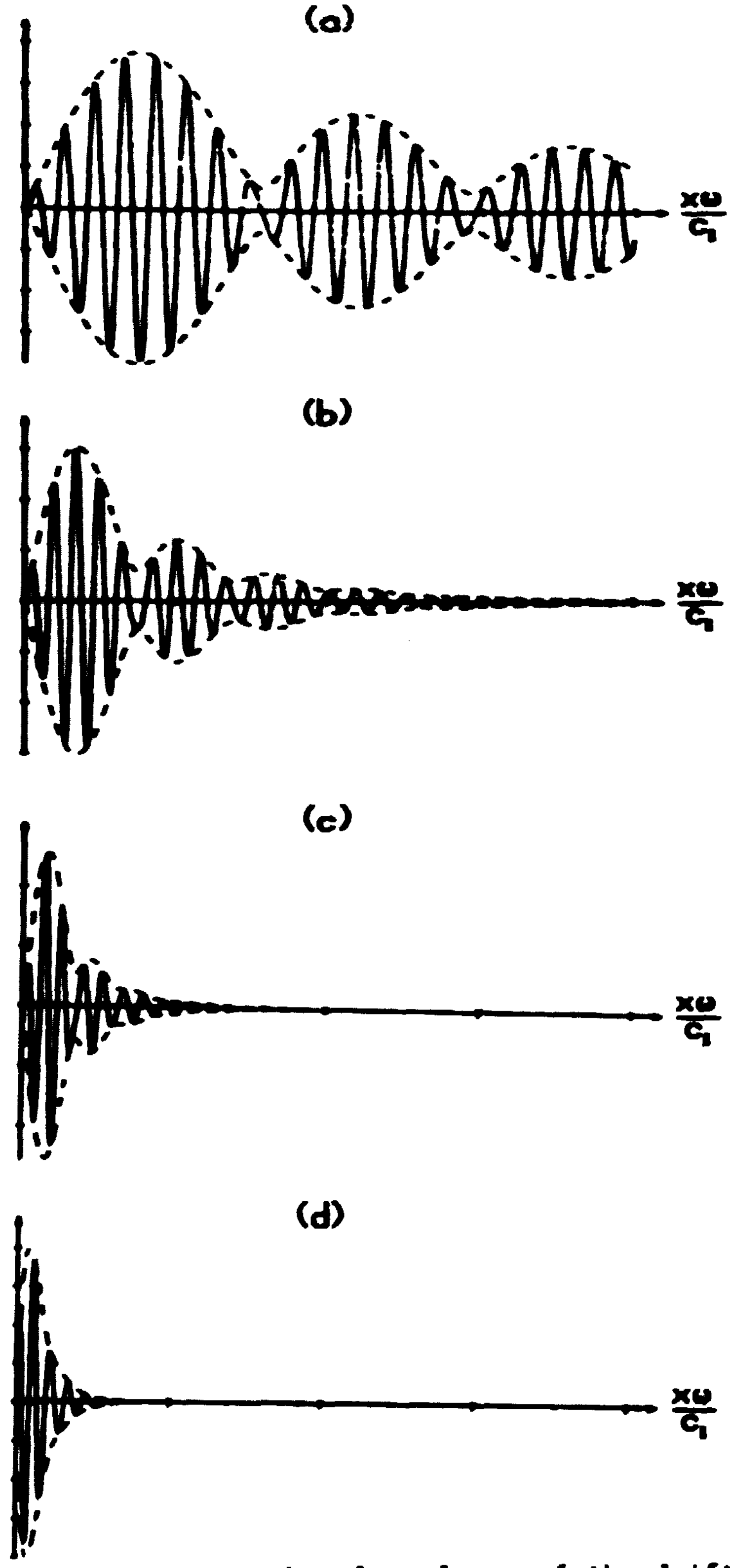

Fig. 5. The wave propagation dependence of the drift velocity. Compare Sato et al. (1977) Fig. 4. $T_i = T_e$, $n_b/n_i = 0.47$, $v_i = 2c_i$, abscissa = 250 $x\omega/c_i$ pr. division. (a) $v_b = 8c_i$, $T_b = 0.09\ T_i$, (b) $v_b = 6c_i$, $T_b = 0.2\ T_i$, (c) $v_b = 4.5c_i$, $T_b = 0.4\ T_i$, and (d) $v_b = 3.5c_i$, $T_b = 0.8\ T_i$.

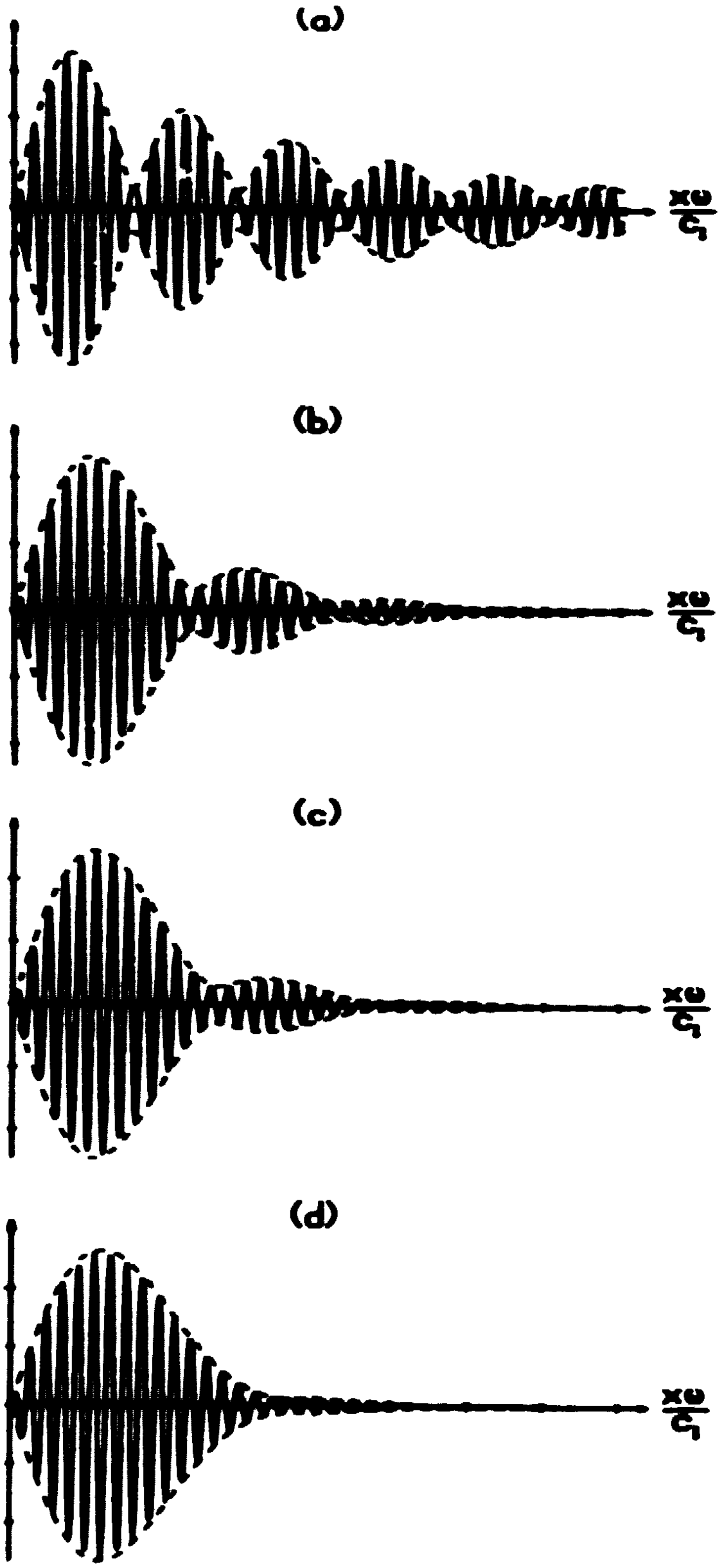

Fig. 6. The wave propagation dependence of the relative beam density. Compare Sato et al. (1977) Fig. 8. $T_b = 0.1\ T_i$, $T_e = T_i$, $v_b = 8.3\ c_i$, $v_i = 2c_i$, abscissa = 250 $x\omega/c_i$ pr. division (a) $n_b/n_i = 0.47$, (b) $n_b/n_i = 0.11$, (c) $n_b/n_i = 0.064$ and (d) $n_b/n_i = 0.015$.